MATH 113 - WORKSHEET 1
WEDNESDAY 6/26

Please work on these problems in 6 groups of 6 or 7 people each. Each group will be assigned one problem, and one member of the group will present their solution to the class once all problems have been solved.

Some problems are harder than others. I will be available to give advice. When you solve your assigned problem, first make sure that everyone in your group understands the solution, then move on to a new problem.

(1) Let X be a set, and define $\text{Fun}(X, X)$ to be the set of all functions $X \to X$. Consider the binary operation of function composition, $\circ$, on $\text{Fun}(X, X)$: Given $f, g : X \to X$, $(f \circ g)$ is the function defined by $(f \circ g)(x) = f(g(x))$.

Does $(\text{Fun}(X, X), \circ)$ possess the following properties? For each, prove it or give a counterexample.

1) Associativity, 2) Identity, 3) Inverses, 4) Commutativity.

(2) How many possible binary operations can be defined on a finite set containing n elements?

(3) Consider the set $\mathbb{R} \times \mathbb{R}$ with the binary operation $\star$ defined by

$$(a, b) \star (c, d) = (a + c, bd).$$

$(\mathbb{R} \times \mathbb{R}, \star)$ is not a group. Why not? Can you remove some elements to make it a group?

(4) Consider the "closeness" relation $\sim$ on $\mathbb{R}$ defined as follows: $x \sim y$ if and only if $|x - y| \leq 1$. Is $\sim$ an equivalence relation? Explain.

(5) Consider a relation R on $\mathbb{Z} \times \mathbb{Z}^*$ (recall $\mathbb{Z}^*$ is the set of all nonzero integers) defined by $(n, d)R(n', d')$ if and only if $nd' = n'd$. Show that R is an equivalence relation.

(6) At some point you may have noticed and/or learned that a number is divisible by 9 if and only if the sum of its digits (in base 10) is divisible by 9.

Examples: $9 \mid 54$, since $5 + 4 = 9$.
$9 \nmid 101$, since $1 + 0 + 1 = 2$.
$9 \mid 3888$ since $3 + 8 + 8 + 8 = 27$.

Explain this phenomenon using modular arithmetic. *Hint:* You can express any n-digit number as $d_0 + d_1 \cdot 10 + d_2 \cdot 10^2 + \cdots + d_n \cdot 10^n$. Then the d_i are its digits in base 10. This is a consequence of the Euclidean algorithm, but you don't have to prove it.

If you have time, try to devise a similar test for divisibility by 11.